

Supervised features selection in MaZda

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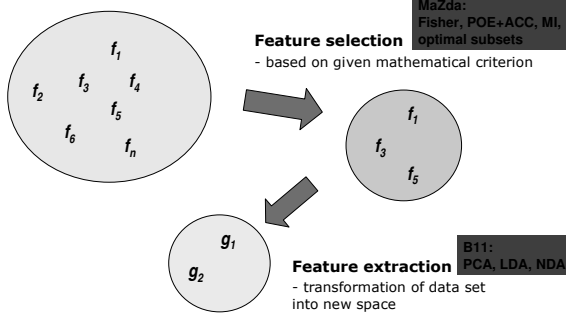
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Why feature reduction?

- it is not known *a priori* which features are best for given texture analysis - one has to consider as many features as possible,
- it is very difficult to manage with almost 300 features generated by **MaZda**,
- large number of features requires large number of data samples (which are not available normally).

Feature reduction

► very important issue of many classification and segmentation tasks

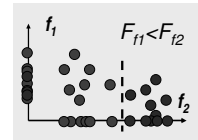


Fisher coefficient

$$F = \frac{D^2}{V^2} = \frac{\frac{1}{K} \sum_{k=1}^K \sum_{j=1}^K P_k P_j |\mu_k - \mu_j|^2}{\sum_{k=1}^K P_k V_k^2}$$

where:

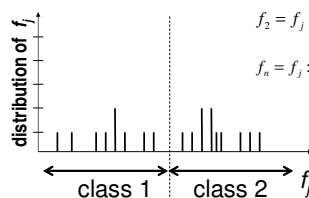
- D - between-class variance,
- V - within-class variance,
- P_k - probability of feature k ,
- V_k, μ_k - variance and mean value of feature k in given class.



MaZda Report window Feature selection->Fisher

| Feature name | V | MSF | MRF | V | MSF | MRF | V | MSF | MRF |
|-----------------|----------|-----------|------------|-----------|----------|----------|------|------|------|
| Volume | 1014 | 1014 | 1014 | 1014 | 1014 | 1014 | 1014 | 1014 | 1014 |
| MeanID | 96.811 | 100.53 | 96.462 | 101.36 | 93.907 | 96.366 | | | |
| Variance3D | 5483.7 | 5131.7 | 5134.1 | 5126.3 | 5144.7 | 5284.4 | | | |
| Skewness3D | 0.01146 | -0.075002 | -0.0040389 | -0.001791 | 0.003011 | 0.01459 | | | |
| Kurtosis3D | -1.4279 | -1.3079 | -1.3648 | -1.3962 | -1.3562 | -1.3816 | | | |
| Pec-01:3D | 1 | 1 | 1 | 1 | 1 | 1 | | | |
| Pec-10:3D | 1 | 1 | 1 | 1 | 1 | 1 | | | |
| Pec-100:3D | 65 | 129 | 65 | 129 | 65 | 65 | | | |
| Pec-30:3D | 193 | 193 | 193 | 193 | 193 | 193 | | | |
| Pec-95:3D | 193 | 193 | 193 | 193 | 193 | 193 | | | |
| Volume45(3D) | 1892 | 1872 | 1872 | 1872 | 1872 | 1872 | | | |
| SI1.0:AngCoMean | 0.062012 | 0.063205 | 0.063891 | 0.063895 | 0.063933 | 0.062796 | | | |
| SI1.0:CoConvst | 10851 | 10867 | 10823 | 8666.7 | 10222 | 10612 | | | |

Probability of Classification Error and Average Correlation Coefficient (POE+ ACC)



$$f_2 = f_j : \min[POE(f_j) + |Corr(f_1, f_j)|]$$

$$f_n = f_j : \min_j [POE(f_j) + \frac{1}{n-1} \sum_{k=1}^{n-1} |Corr(f_k, f_j)|]$$

$$POE(f_j) = \frac{\text{number of misclassified samples}}{\text{total number of samples}} = \frac{2}{20}$$

$$f_1 = f_j : \min_j [POE(f_j)]$$

MaZda Report window Feature selection->POE+ACC

10 features with the lowest POE+ACC coefficient

| Feature name | MFE | MSE | MSE | MSE | MSE | MSE | MSE |
|-----------------|----------|----------|-----------|-----------|----------|----------|------|
| Volume | 1014 | 1014 | 1014 | 1014 | 1014 | 1014 | 1014 |
| MeanSD | 102.11 | 100.03 | 96.306 | 96.83 | 91.699 | 95.527 | |
| VarianceSD | 5215 | 5286.6 | 5216.5 | 5157 | 5184.4 | 4953.3 | |
| SkewnessSD | -0.09763 | -0.08982 | 0.0043082 | -0.025347 | 0.094905 | 0.033678 | |
| KurtosisSD | -1.3789 | -1.3897 | -1.384 | -1.3692 | -1.3612 | -1.3193 | |
| Pec 01SD | 1 | 1 | 1 | 1 | 1 | 1 | |
| Pec 10SD | 1 | 1 | 1 | 1 | 1 | 1 | |
| Pec 50SD | 129 | 129 | 65 | 129 | 65 | 65 | |
| Pec 95SD | 193 | 193 | 193 | 193 | 193 | 193 | |
| VolumeSI(0,0) | 1872 | 1872 | 1872 | 1872 | 1872 | 1872 | |
| SI(0,0)ngfcHom | 0.063936 | 0.06458 | 0.063739 | 0.063233 | 0.064646 | 0.064411 | |
| SI(0,0)Contrast | 10218 | 9789.5 | 10577 | 9670.5 | 9045.3 | 8877.8 | |

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Mutual Information (MI)

MI between two random variables F and C is estimated by:

$$I(F, C) = \sum_f \sum_c P(F, C) \log_2 \frac{P(F, C)}{P(F)P(C)}$$

where the F and C are quantized.

MI=0 when F (feature) and C (class category) are independent (incorrect classification for all f_i).

$$class(f_i) \neq c_i \text{ for all } i$$

Large value of MI means that given feature carries information about class membership (F and C are correlated).

$$class(f_i) = c_i \text{ for some } i$$

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MaZda Report window Feature selection->MI

10 features with the highest Mutual Information coefficient

| Feature name | MFE | MSE | MSE | MSE | MSE | MSE | MSE |
|-----------------|----------|----------|-----------|-----------|----------|----------|------|
| Volume | 1014 | 1014 | 1014 | 1014 | 1014 | 1014 | 1014 |
| MeanSD | 102.11 | 100.03 | 96.306 | 96.83 | 91.699 | 95.527 | |
| VarianceSD | 5215 | 5286.6 | 5216.5 | 5157 | 5184.4 | 4953.3 | |
| SkewnessSD | -0.09763 | -0.08982 | 0.0043082 | -0.025347 | 0.094905 | 0.033678 | |
| KurtosisSD | -1.3789 | -1.3897 | -1.384 | -1.3692 | -1.3612 | -1.3193 | |
| Pec 01SD | 1 | 1 | 1 | 1 | 1 | 1 | |
| Pec 10SD | 1 | 1 | 1 | 1 | 1 | 1 | |
| Pec 50SD | 129 | 129 | 65 | 129 | 65 | 65 | |
| Pec 95SD | 193 | 193 | 193 | 193 | 193 | 193 | |
| VolumeSI(0,0) | 1872 | 1872 | 1872 | 1872 | 1872 | 1872 | |
| SI(0,0)ngfcHom | 0.063936 | 0.06458 | 0.063739 | 0.063233 | 0.064646 | 0.064411 | |
| SI(0,0)Contrast | 10218 | 9789.5 | 10577 | 9670.5 | 9045.3 | 8877.8 | |

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Optimal subset feature selection (brute force method)

In this method all feature subsets are analysed. Thus it is possible to find a subset which provides a minimal classification error (using 1-NN classifier). The number of subsets is defined by:

$$S_{Tot} = \sum_{i=1}^n \binom{n}{i}$$

where n - total number of features.

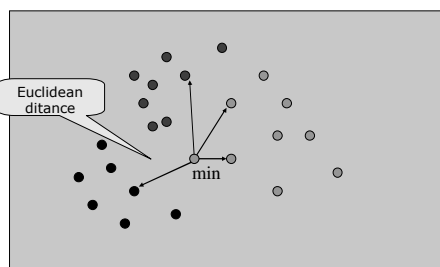
In MaZda 4.0 $n=284$, then $S_{Tot} > 1e70$

Thus evaluation of subsets with only 1, 2 and 3 features is implemented:

- single features: 284 subsets
- feature pairs: 40186 subsets
- feature triples: 3777484 subsets

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1-Nearest Neighbour Classifier, 1-NN)



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MaZda Report window Feature selection->Optimal subsets->Pairs

10 feature pairs with the lowest 1-NN classification error

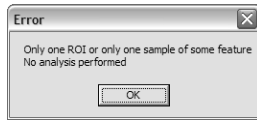
| Feature name | MFE | MSE | MSE | MSE | MSE | MSE | MSE |
|-----------------|----------|----------|-----------|-----------|----------|----------|------|
| Volume | 1014 | 1014 | 1014 | 1014 | 1014 | 1014 | 1014 |
| MeanSD | 102.11 | 100.03 | 96.306 | 96.83 | 91.699 | 95.527 | |
| VarianceSD | 5215 | 5286.6 | 5216.5 | 5157 | 5184.4 | 4953.3 | |
| SkewnessSD | -0.09763 | -0.08982 | 0.0043082 | -0.025347 | 0.094905 | 0.033678 | |
| KurtosisSD | -1.3789 | -1.3897 | -1.384 | -1.3692 | -1.3612 | -1.3193 | |
| Pec 01SD | 1 | 1 | 1 | 1 | 1 | 1 | |
| Pec 10SD | 1 | 1 | 1 | 1 | 1 | 1 | |
| Pec 50SD | 129 | 129 | 65 | 129 | 65 | 65 | |
| Pec 95SD | 193 | 193 | 193 | 193 | 193 | 193 | |
| VolumeSI(0,0) | 1872 | 1872 | 1872 | 1872 | 1872 | 1872 | |
| SI(0,0)ngfcHom | 0.063936 | 0.06458 | 0.063739 | 0.063233 | 0.064646 | 0.064411 | |
| SI(0,0)Contrast | 10218 | 9789.5 | 10577 | 9670.5 | 9045.3 | 8877.8 | |

Time consuming (pairs, triples) !!!

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Automatic feature selection (Fisher, POE+ACC, MI, Optimal subsets)

- it is necessary to define at least two classes (two ROIs with different names),
- each class must contain at least two samples



It is also possible to perform manual feature selection (by selecting/deselecting features in Report window)

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Analysis examples:

Feature selection:

- four methods were used to obtain best features
- analysis of feature subsets was performed

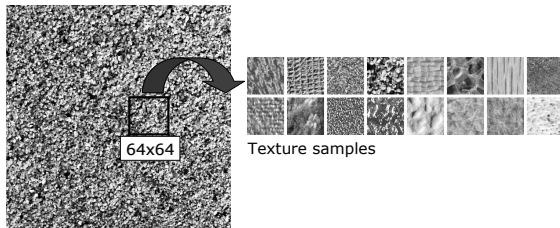
Classification (1NN - one nearest neighbor):

- raw data
- data after linear discriminant analysis (LDA)

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Experimental Data #1

- 16 textures from Brodatz album, 512x512 pixels
- each image was divided into 64 squares (64x64)
- 1024 samples of 16 texture classes altogether
- for each sample 269 texture features have been calculated using MaZda software ($\pm 3\sigma$ norm.) (gradient matrix, co-occurrence matrix, RL matrix, wavelet, AR model)



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1-NN classification results for 4 features selected using different methods

| optimal subset | MI | | Fisher | | POE | |
|--|-----|-----|--------|-----|-----|-----|
| | raw | LDA | raw | LDA | raw | LDA |
| | 0 | 16 | 8 | 79 | 59 | 145 |
| Number of misclassified samples (total no. of samples: 1024) | | | | | | |

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Optimal feature subset:
Sigma, MinNorm, GrMean, S(0,2)Correlat
1-NN error: 0

MI:

Sigma
MinNorm
S(1,0)Contrast
Perc.10%
S(0,1)DifVarnc
S(0,2)DifVarnc
GrMean
S(0,2)Correlat
S(0,3)Contrast
S(1,0)Correlat

1-NN error: 0

Fisher:

S(0,2)DifEntrp
S(0,3)DifEntrp
S(0,1)DifEntrp
Sigma
S(0,2)Contrast
Variance
S(0,4)DifEntrp
Perc.10%
MinNorm
S(0,2)Correlat

1-NN error: 0

POE:

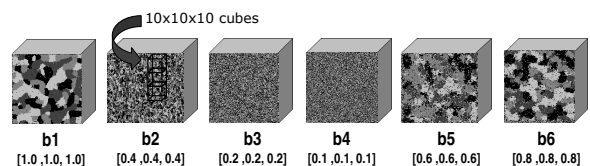
S(0,2)DifEntrp
Teta4
S(0,5)SumVarnc
S(5,0)SumVarnc
MaxNorm
WavEnHH_s-2
S(0,2)AngScMom
Teta3
GrNonZeros
Perc.10%

1-NN error: 26
1-NN error after LDA: 23

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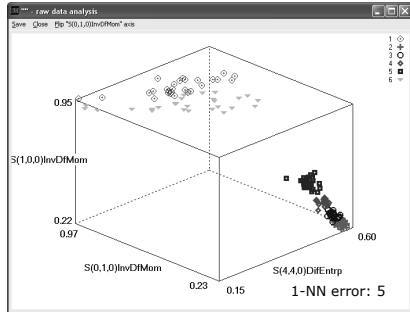
Experimental Data #2

- 6 3D textures generated using MRF model, 130x130x130 pixels
- each image was divided into 27 squares (10x10x10)
- 162 samples of 6 texture classes altogether
- for each sample 284 texture features have been calculated using MaZda software (histogram, 3D co-occurrence matrix)

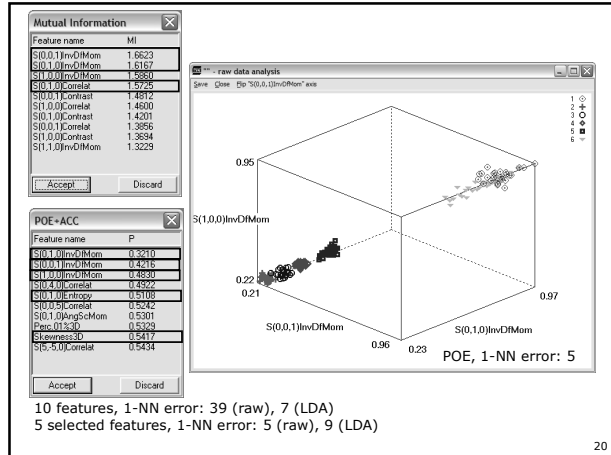


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Optimal subset with 3 features:
 $S(1,0,0)$ InvDfMom, $S(0,1,0)$ InvDfMom,
 $S(4,4,0)$ DifEntrp



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Conclusions

- None of the three feature selection methods was able to select optimal subset of features. However some of them found features with the same classification error.
- Quality of feature selection method depends on classification task (data #1 – MI; data #2 – POE).
- Feature subsets may provide better classification than the whole set (in case of Fisher, POE, MI) using 1-NN classifier.

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Exercise 1

- Start MaZda or, if it is already started, close all the report tab-pages
- Load image *texture1.bmp* from *Tutorials\3a_Texture_analysis_1* folder
- Switch to the overlapping mode and create 16 regions of interest of an approximate size 35x35 pixels (observe the status bar for a size information) with *Draw rectangle* tool
- Set analysis options and run the analysis
- In a Report window set class name *Biology* for all the columns
- Load image *texture2.bmp* from *Tutorials\3a_Texture_analysis_1* folder
- Run the analysis (use the same regions and analysis options as previously)
- In a Report window set class name *Mineral* for all the columns of the second tab-page
- Find features that may be useful for discrimination of the two classes

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Exercise 1 (continued)

- Perform automatic feature selection with Fisher criterion using **Feature selection->Fisher**
- Save selected features to a file *fisher.sel* using **File->Save selected**
- Run **b11** using **Tools->B11 analysis**
- Observe **b11** options window selecting **Options**
- Select only three top features in Feature selection box
- Observe feature distribution using **Analysis->Raw data**
- Perform PCA and LDA analysis and observe feature distribution in the new data space

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