Technical University of Lodz
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Deformable mesh for regularization of three-dimensional image registration

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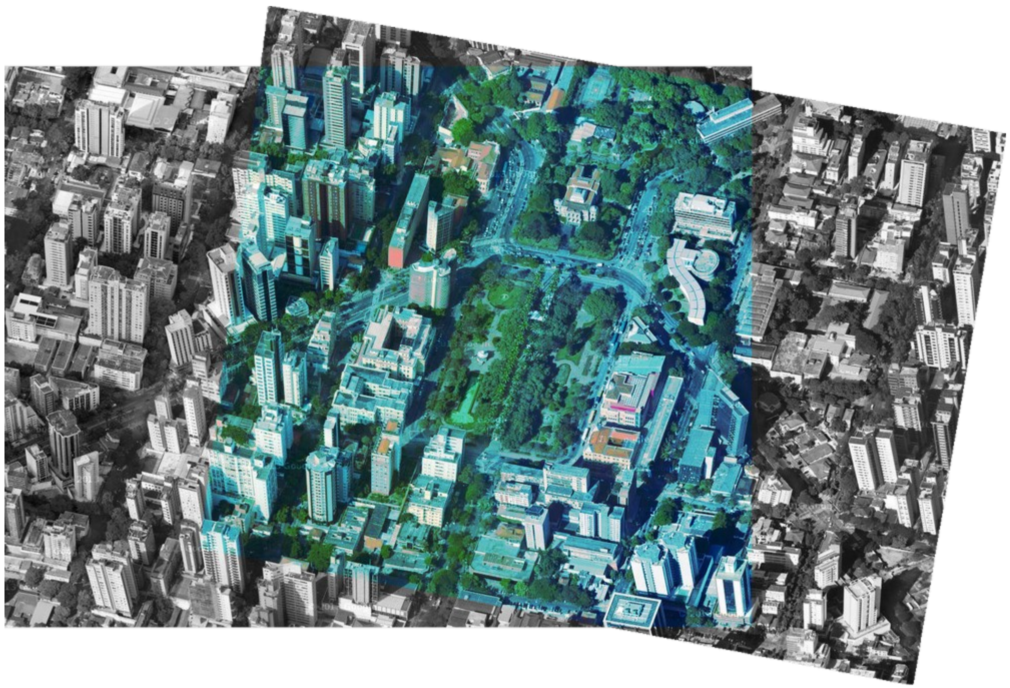
Image registration

Image registration is a process of fitting together a content of two or more images. In medical applications it is required to correctly overlap or integrate data obtained by various imaging modalities. Another use of the registration is to follow specific anatomical structures in a series of images acquired in time.



[wordpress.com, Introduction-to-non-linear-optimization-for-image-registration]

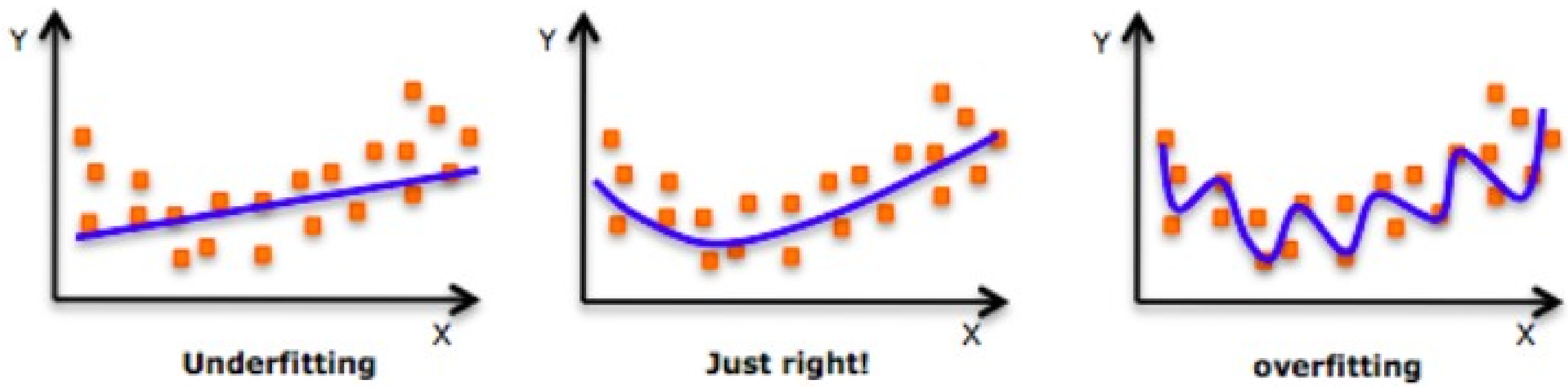
Points pairing

Pairing or finding corresponding points, involves comparison of the point's neighborhoods in terms of some similarity measure. Localization of such points is usually inaccurate due to image discrete form, noise or repeated image patterns.Therefore, the image registration is indeed an inverse and ill-posed problem, which requires regularization.

[Geordie Rose, ResearchGate]

Regularization

In ill-posed problems, particularly when solving inverse problems, the unique solution may not exist. We usually deal with a number or a range of good or sub-optimal solutions. **Regularization** is a way of introducing additional information that enables narrowing the range of solutions or indicate the one which is optimal.

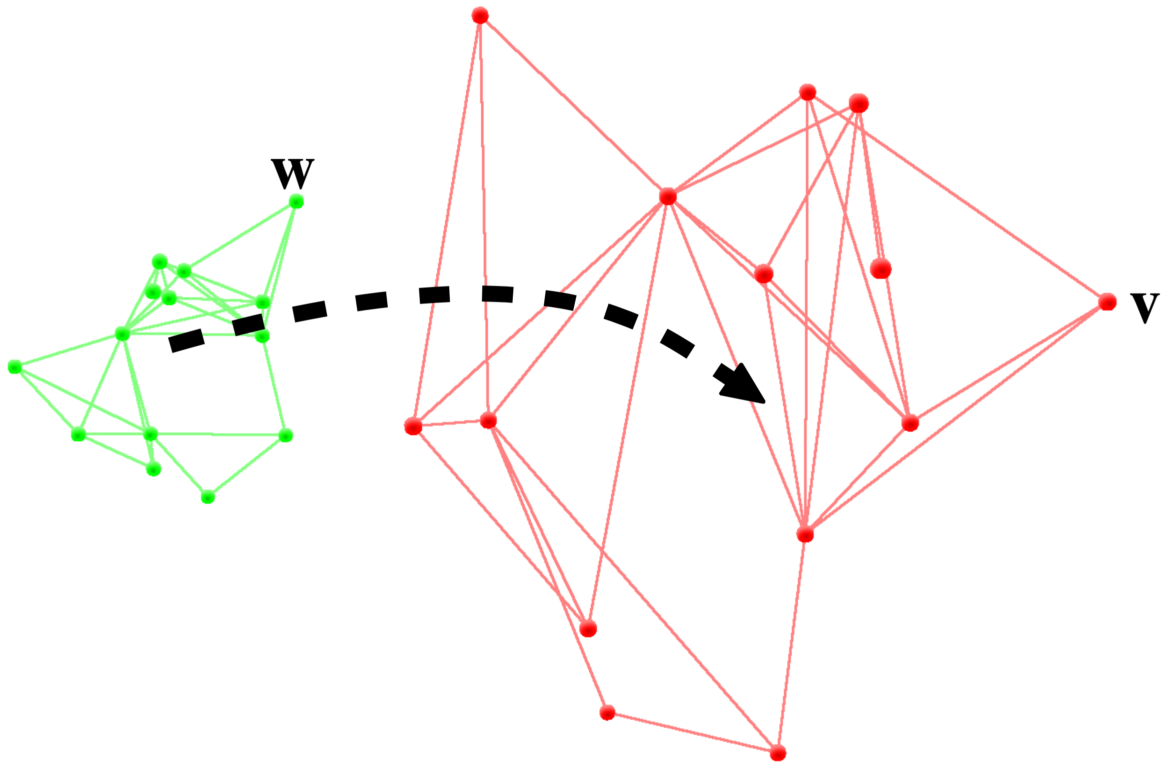


[Analytics Vidhya, An Overview of Regularization Techniques in Deep Learning]

Regularization in registration

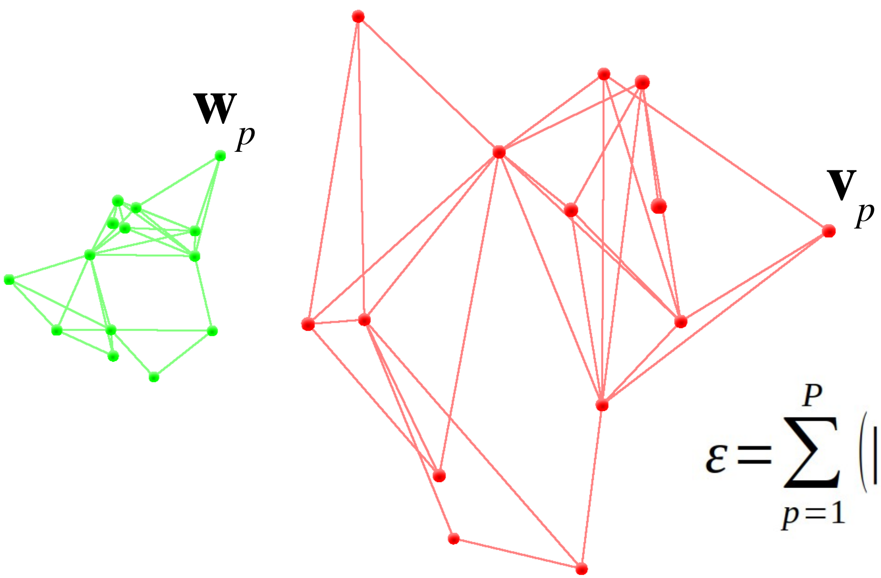
1. Both the images present the same rigid structure – transformation is limited to rotation and translation.
2. Misalignment between the images can be corrected by an affine transformation. In addition the affine transformation enables resizing and shear.
3. Local image deformation is enabled – curved or elastic image transform. Polynomials of limited degree are often used.
4. No regularization – assumes that paired points are perfectly identified. One image is warped to exactly align its points with their counterparts in the other image. The b-splines may be used to manage warping and interpolation.

Transformation from a set of paired points



$$\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_D \end{bmatrix} \approx \begin{bmatrix} j_{11} & \cdots & j_{1D} \\ & \vdots & \\ j_{D1} & \cdots & j_{DD} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_D \end{bmatrix} + \begin{bmatrix} t_1 \\ \vdots \\ t_D \end{bmatrix} = \mathbf{J}\,\mathbf{w} + \mathbf{T}$$

Transformation from a set of paired points



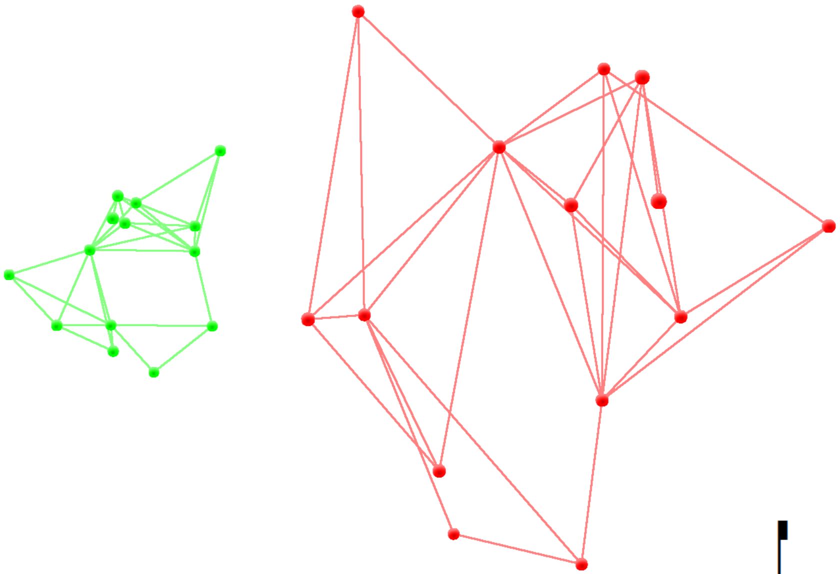
$$\varepsilon=\sum_{p=1}^{P}\left(\left|\mathbf{J}\,\mathbf{w}_p+\mathbf{T}-\mathbf{v}_p\right|^2\right)=\sum_{p=1}^{P}\left(\sum_{n=1}^{D}\left(\sum_{m=1}^{D}\left(j_{nm}w_{pm}\right)+t_n-v_{pn}\right)^2\right)$$

P – number of points
D – number of dimenssions

$$\frac{\partial\varepsilon}{\partial t_k}=2\sum_{p=1}^{P}\left(t_k+\sum_{m=1}^{D}\left(j_{km}w_{pm}\right)-v_{pk}\right)=0$$

$$\frac{\partial\varepsilon}{\partial j_{kl}}=2\sum_{p=1}^{P}\left(w_{pl}\sum_{m=1}^{D}\left(w_{pm}j_{km}\right)+w_{pl}t_k-w_l v_{pk}\right)=0$$

Transformation from a set of paired points



$$\begin{bmatrix} j_{k1} \\ \vdots \\ j_{kD} \\ t_k \end{bmatrix} = \begin{bmatrix} \sum_{p=0}^{P} w_{p1}^{2} & \cdots & \sum_{p=0}^{P} w_{p1} w_{pD} & \sum_{p=0}^{P} w_{p1} \\ \vdots & & \vdots & \vdots \\ \sum_{p=0}^{P} w_{p1} w_{pD} & \cdots & \sum_{p=0}^{P} w_{pD}^{2} & \sum_{p=0}^{P} w_{pD} \\ \sum_{p=0}^{P} w_{p1} & \cdots & \sum_{p=0}^{P} w_{pD} & P \end{bmatrix}^{-1} \begin{bmatrix} \sum_{p=0}^{P} w_{p1} v_{pk} \\ \vdots \\ \sum_{p=0}^{P} w_{pD} v_{pk} \\ \sum_{p=0}^{P} v_{pk} \end{bmatrix}$$

Transformation from a set of paired points

$$\mathbf{J} = \mathbf{U}\mathbf{S} = \mathbf{U}\mathbf{Q}^{-1}\mathbf{D}\mathbf{Q} = |\mathbf{D}|\mathbf{U}\mathbf{Q}^{-1}\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_D \end{bmatrix}\mathbf{Q}$$

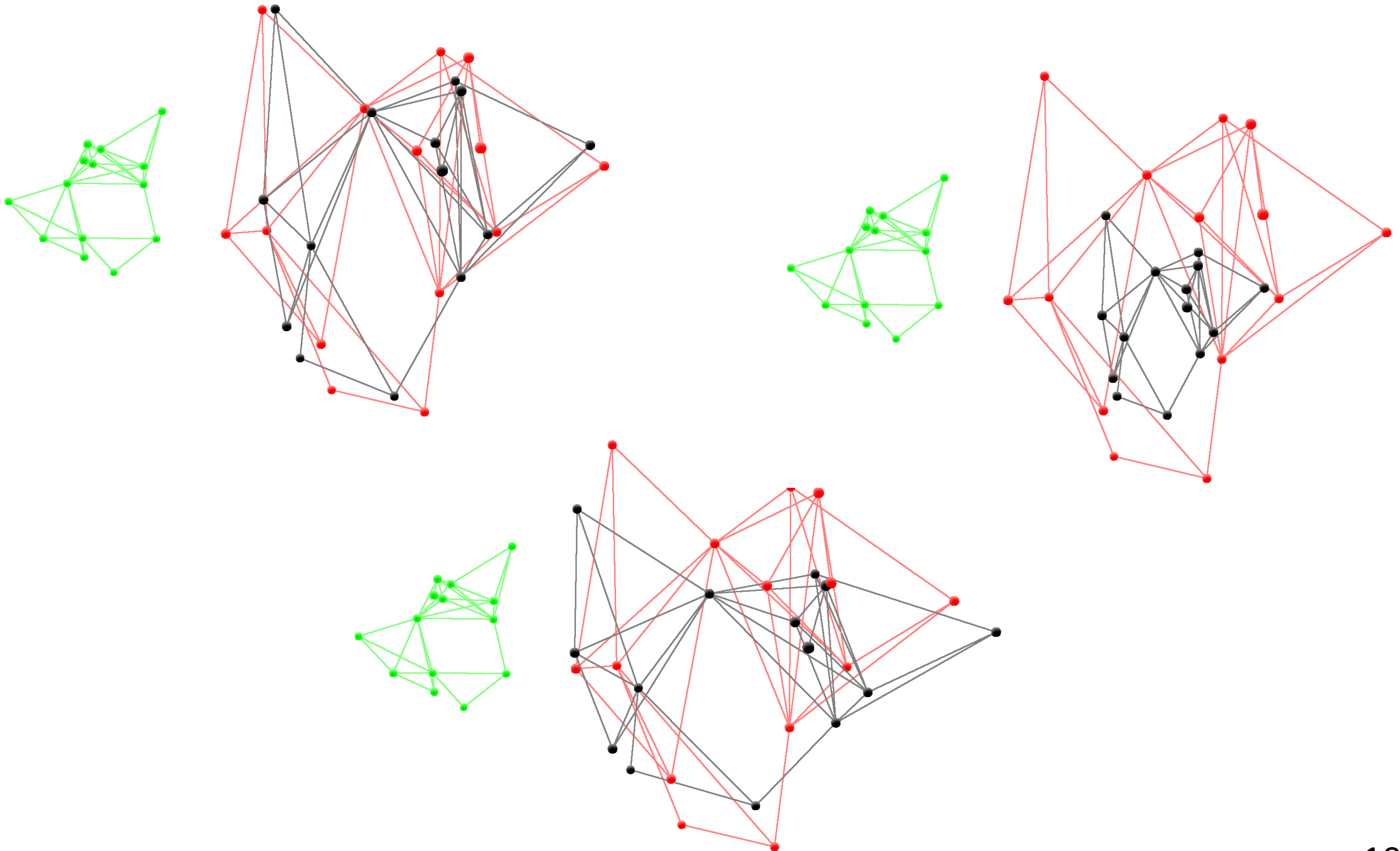
U – orthogonal matrix (rotation)
S – symmetric matrix
Q – orthogonal matrix (shear orientation)
D – diagonal matrix (shear proportions)
|**D**| – determinant of **D** (resizing)

Affin $\mathbf{J} = |\mathbf{D}|\mathbf{U}\mathbf{Q}^{-1}\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_D \end{bmatrix}\mathbf{Q}$

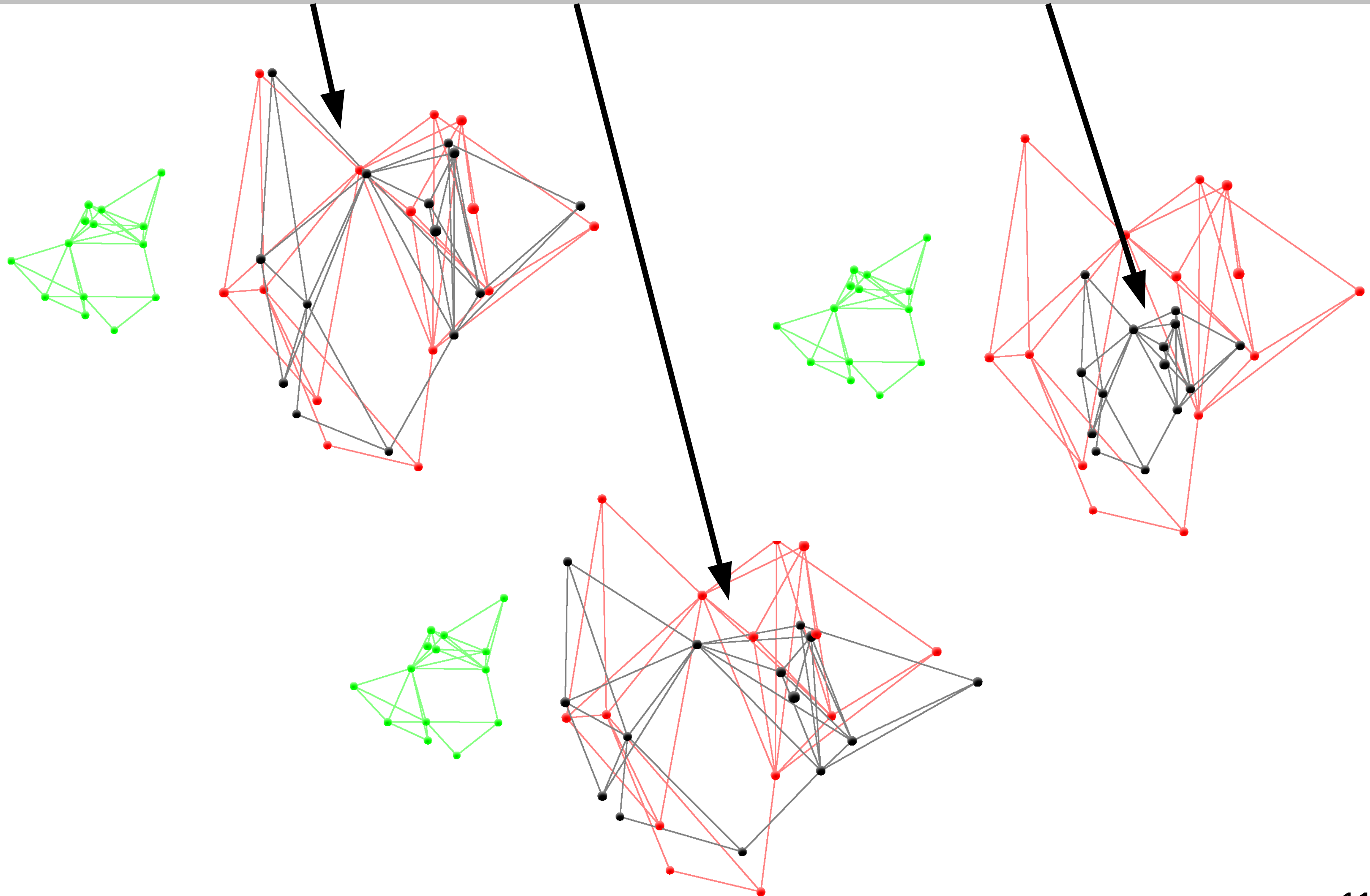
Procrustes (approximation) $\mathbf{J} = |\mathbf{D}|\mathbf{U}\cancel{\mathbf{Q}^{-1}}\cancel{\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_D \end{bmatrix}}\cancel{\mathbf{Q}}$

Unimodal (volume preserving) $\mathbf{J} = \cancel{|\mathbf{D}|}\mathbf{U}\mathbf{Q}^{-1}\begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_D \end{bmatrix}\mathbf{Q}$

Affine, Procrustes or Unimodal?



Affine, Procrustes or Unimodal?



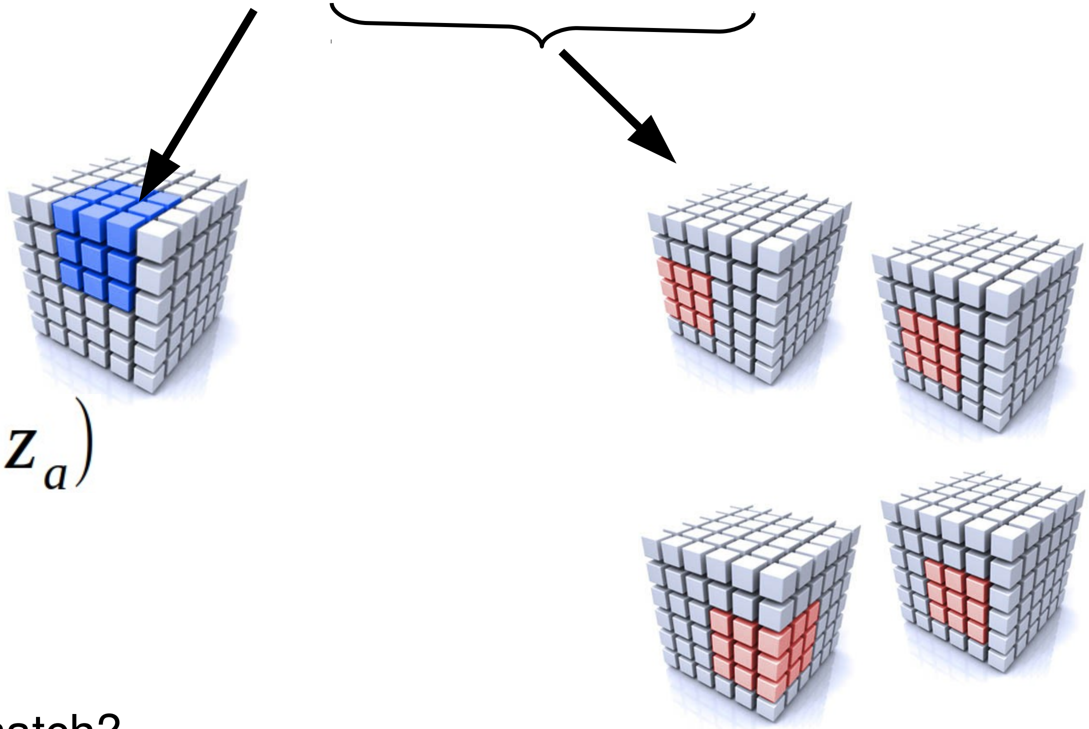
Comparison

	Translation	Rotation	Shear	Scaling	Bending
Affine	**+**	**+**	**+**	**+**	**–**
Procrustes	**+**	**+**	**–**	**+**	**–**
Unimodal	**+**	**+**	**+**	**–**	**–**
Curved (polynomials)	**+**	**+**	**+**	**+**	**+**
Madical image registration	**+**	**+/c**	**+/c**	**–/c**	**+/c**

+ enabled
– disabled
c control required

Image similarity measures

$$M_a(x_b, y_b, z_b)$$



$$I_A(x_a, y_a, z_a)$$

$$I_B(x_b, y_b, z_b)$$

How to find best match?

1. Search over all the image.
2. Search some neighborhood of *a* in *B*?
3. Use gradient descent.

Image similarity/disimilarity measures

MAD – mean absolute difference – the same modality, brightness and contrast

$$MAD_a(x_b, y_b, z_b) = \frac{1}{(2R+1)^3} \sum_{i=-R}^{R} \sum_{j=-R}^{R} \sum_{k=-R}^{R} |I_A(x_a+i, y_a+j, z_a+k) - I_B(x_b+i, y_b+j, z_b+k)|$$

NCM – normalized covariance measure or normalized cross-correlation – the same modality, brightness and contrast missmatch

$$NCM_a(x_b, y_b, z_b) = \frac{\sum_{i=-R}^{R} \sum_{j=-R}^{R} \sum_{k=-R}^{R} \left(I_A(x_a+i, y_a+j, z_a+k) - \mu_a\right)\left(I_B(x_b+i, y_b+j, z_b+k) - \mu_b\right)}{\sqrt{\sum_{i=-R}^{R} \sum_{j=-R}^{R} \sum_{k=-R}^{R} \left(I_A(x_a+i, y_a+j, z_a+k) - \mu_a\right)^2} \sqrt{\sum_{i=-R}^{R} \sum_{j=-R}^{R} \sum_{k=-R}^{R} \left(I_B(x_b+i, y_b+j, z_b+k) - \mu_b\right)^2}}$$

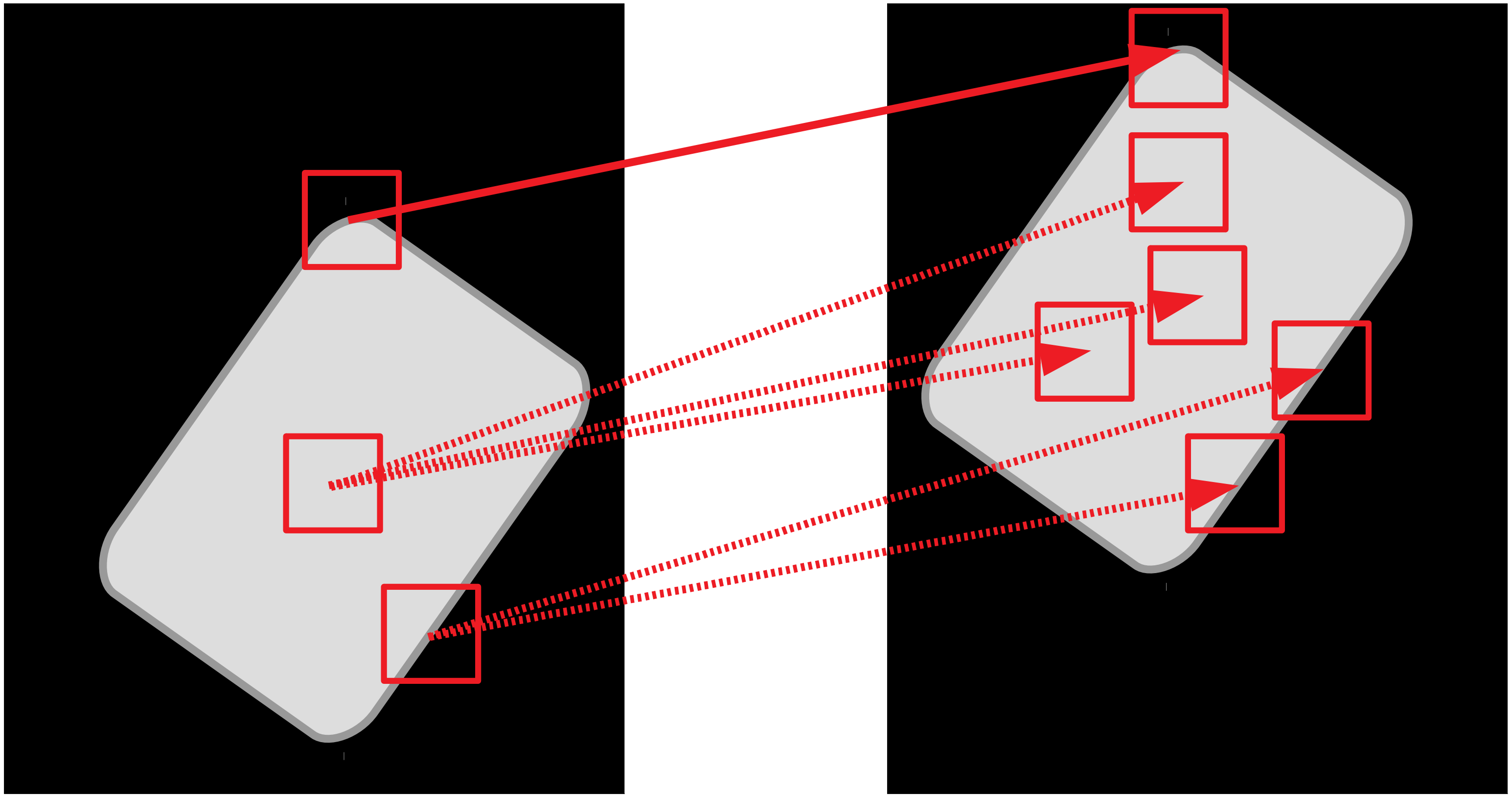
MI – mutual information – differing modalities registration

$$MI_a(x_b, y_b, z_b) = \sum_{g_A=0}^{I_m} \sum_{g_B=0}^{I_m} P(g_A, g_B) \log \frac{P(g_A, g_B)}{P(g_A) P(g_B)}$$

$$P(g_A) = \frac{1}{(2R+1)^3} \sum_{i=-R}^{R} \sum_{j=-R}^{R} \sum_{k=-R}^{R} \begin{cases} 1, & I_A(x_a+i, y_a+j, z_a+k) = g_A \\ 0, & I_A(x_a+i, y_a+j, z_a+k) \neq g_A \end{cases}$$

$$P(g_A, g_B) = \frac{1}{(2R+1)^3} \sum_{i=-R}^{R} \sum_{j=-R}^{R} \sum_{k=-R}^{R} \begin{cases} 1, & I_B(x_b+i, y_b+j, z_b+k) = g_B \land I_A(x_a+i, y_a+j, z_a+k) = g_A \\ 0, & I_B(x_b+i, y_b+j, z_b+k) \neq g_B \lor I_A(x_a+i, y_a+j, z_a+k) \neq g_A \end{cases}$$

Feature points



Deformable model inspiration

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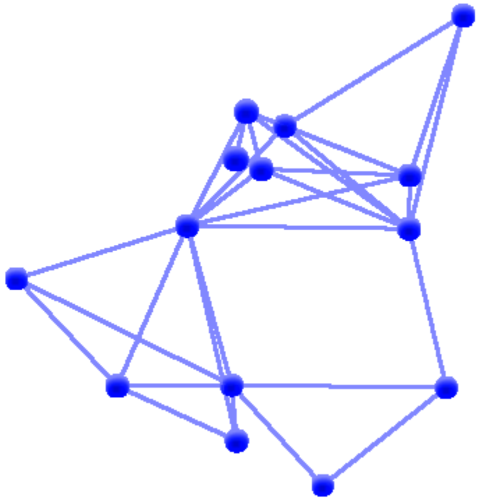

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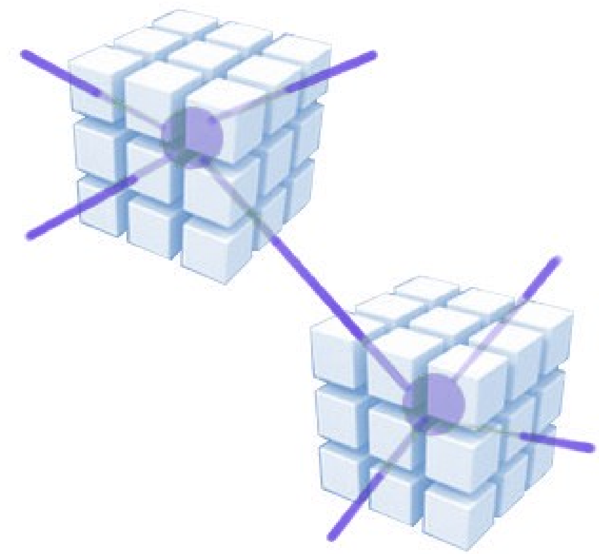


$$E=\rho\sum_{p=1}^{P} M_p(x,y,z)+\xi\varepsilon$$

$$\varepsilon=\sum_{p=1}^{P}\left(\left|\mathbf{J}\,\mathbf{w}_p+\mathbf{T}-\mathbf{v}_p\right|^2\right)$$

Deformable model inspiration

$$E = \rho \sum_{p=1}^{P} M_p(x, y, z) + \xi \varepsilon$$

$$\mathbf{v}_k^{(i+1)} = \mathbf{v}_k^{(i)} - \nabla\left(\rho M_k^{(i)} + \xi \varepsilon_k^{(i)}\right)$$

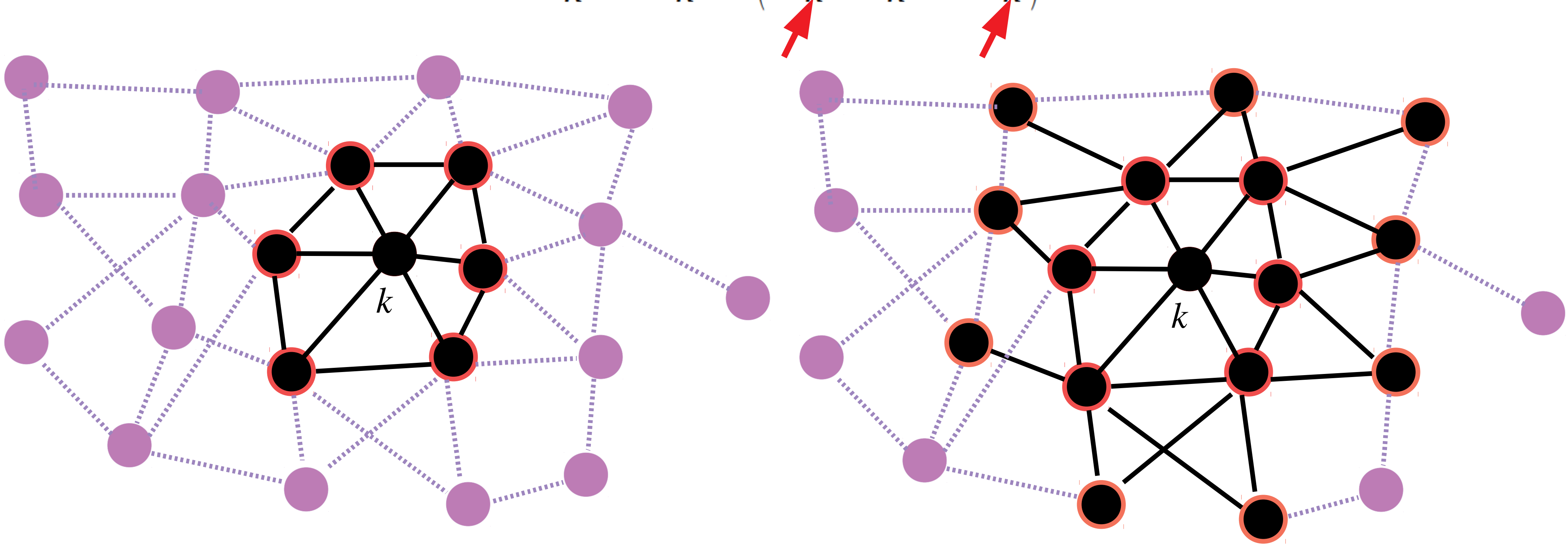
$$\nabla M_k = \nabla NCM_k([x],[y],[z]) = \begin{bmatrix} NCM_k([x]+1,[y],[z]) - NCM_k([x]-1,[y],[z]) \\ NCM_k([x],[y]+1,[z]) - NCM_k([x],[y]-1,[z]) \\ NCM_k([x],[y],[z]+1) - NCM_k([x],[y],[z]-1) \end{bmatrix}$$

$$\varepsilon_k = \left|\mathbf{v}_k - \left(\mathbf{J}_k \mathbf{w}_k + \mathbf{T}_k\right)\right|^2$$

$$\nabla \varepsilon_k = \mathbf{v}_k - \left(\mathbf{J}_k \mathbf{w}_k + \mathbf{T}_k\right)$$

Bending

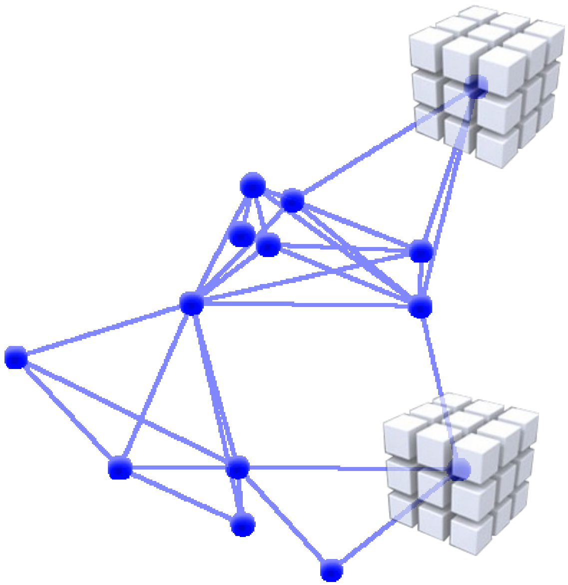
$$\nabla \varepsilon_k = \mathbf{v}_k - \left(\mathbf{J}_k \mathbf{w}_k + \mathbf{T}_k \right)$$



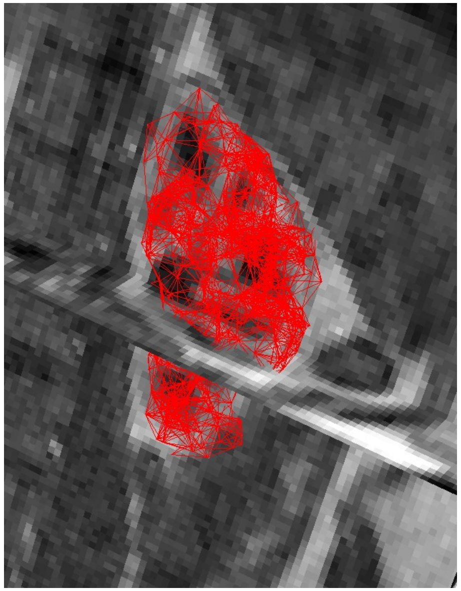
The transformation matrix and translation vector is estimated locally for some neighborhood of node (point) *k*. Applying narrow neighborhoods enable bending of the whole structure. Applying wider neighborhoods restrict bending.

Properties

1. Irregular grids and matching of feature points



2. Selective registration of image fragments



Properties

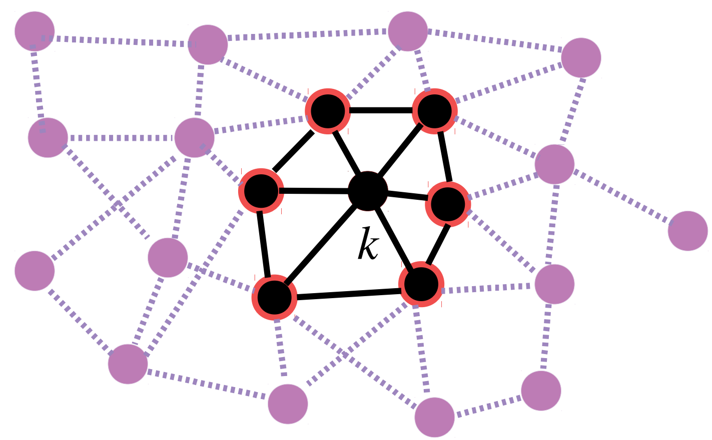
3. Balance between regularization and image terms

$$E = \rho \sum_{p=1}^{P} M_p(x, y, z) + \xi \varepsilon$$

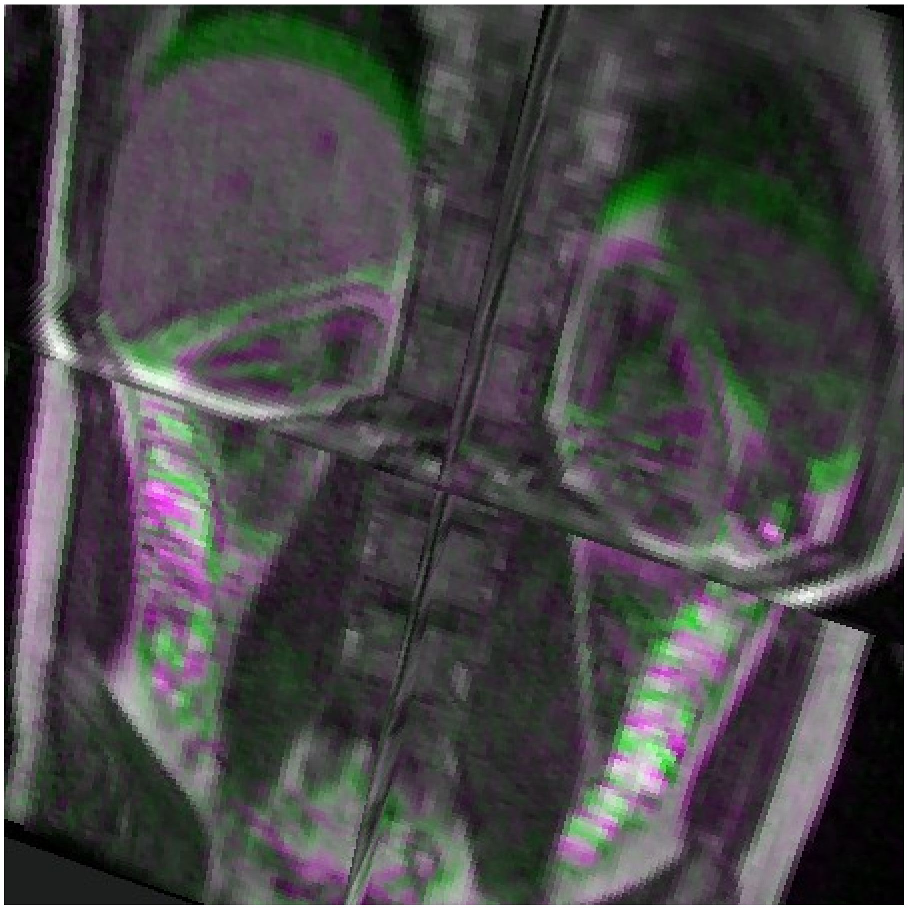
4. Freedom to include or exclude transformation components

$$\mathbf{J} = |\mathbf{D}| \, \mathbf{U} \mathbf{Q}^{-1} \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_D \end{bmatrix} \mathbf{Q}$$

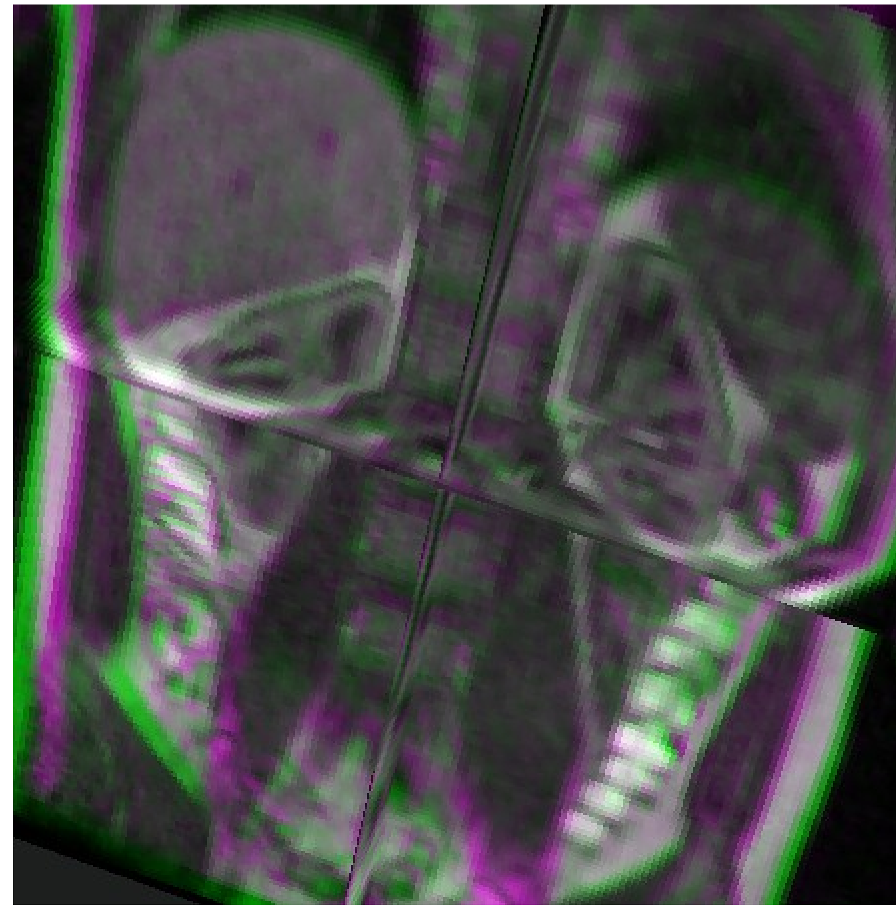
5. Control over bending



Applications

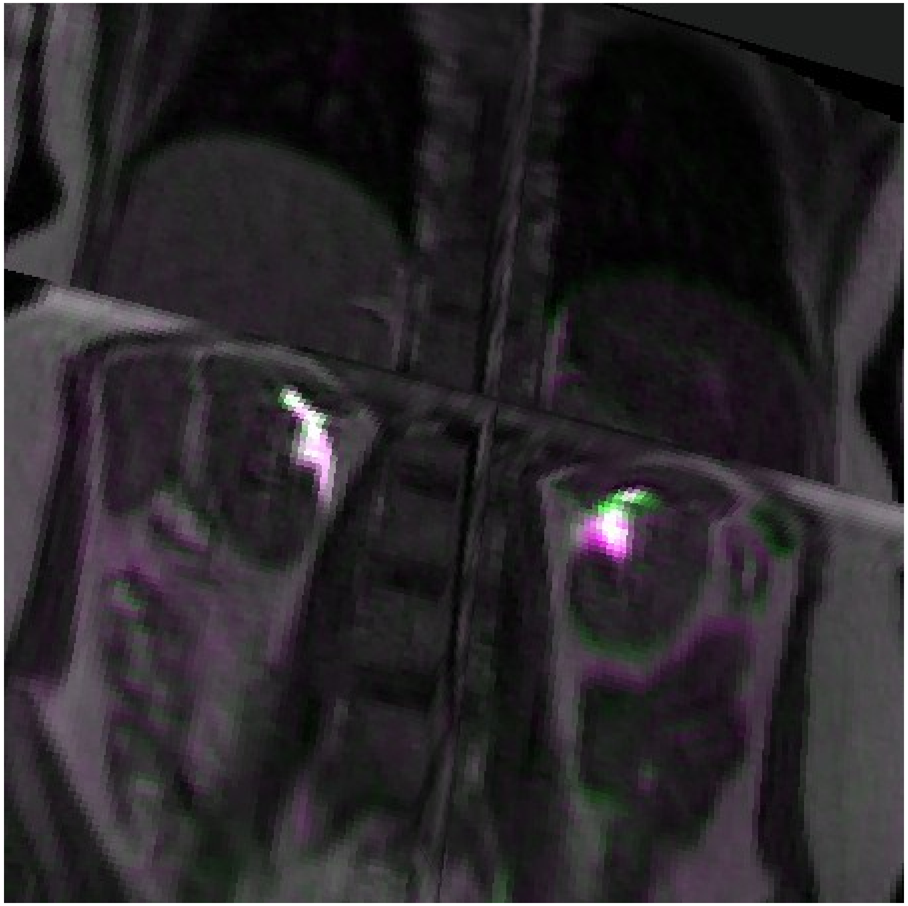


Before

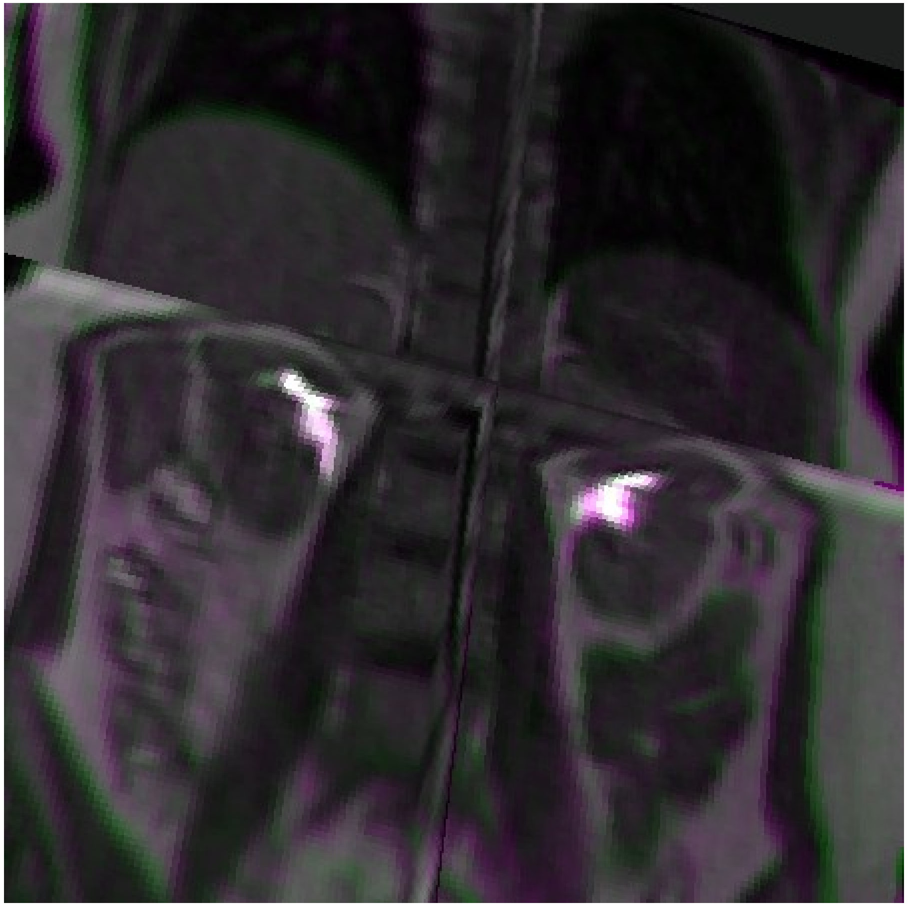


After

Applications

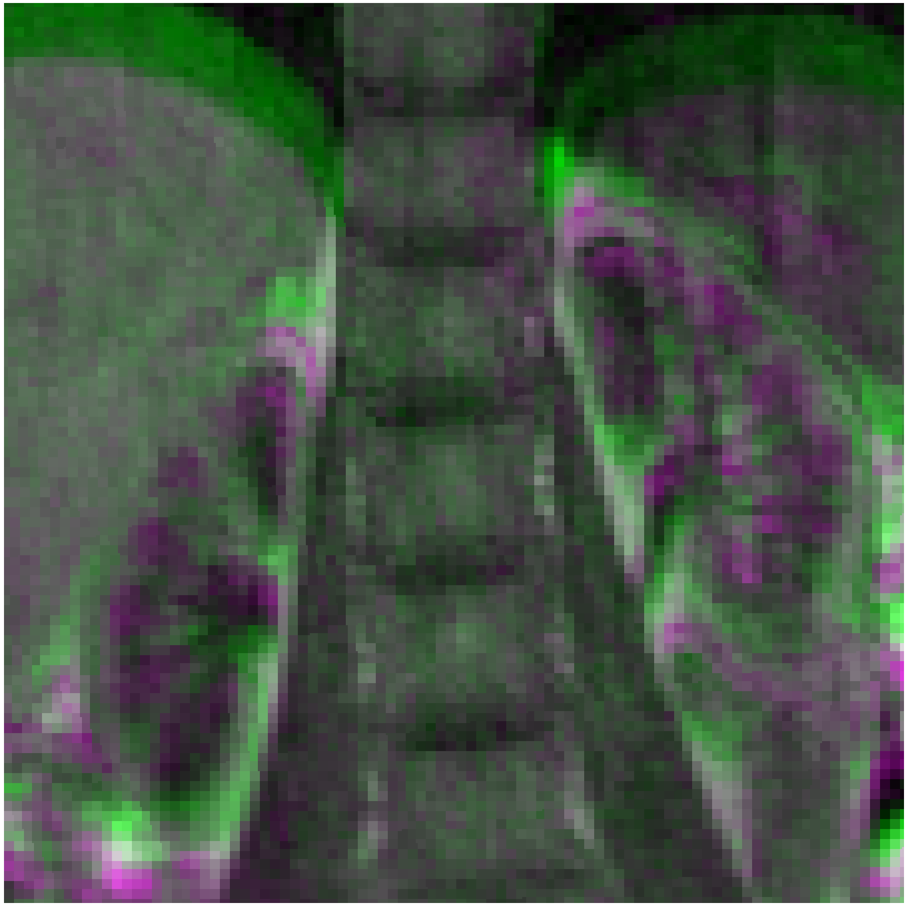


Before

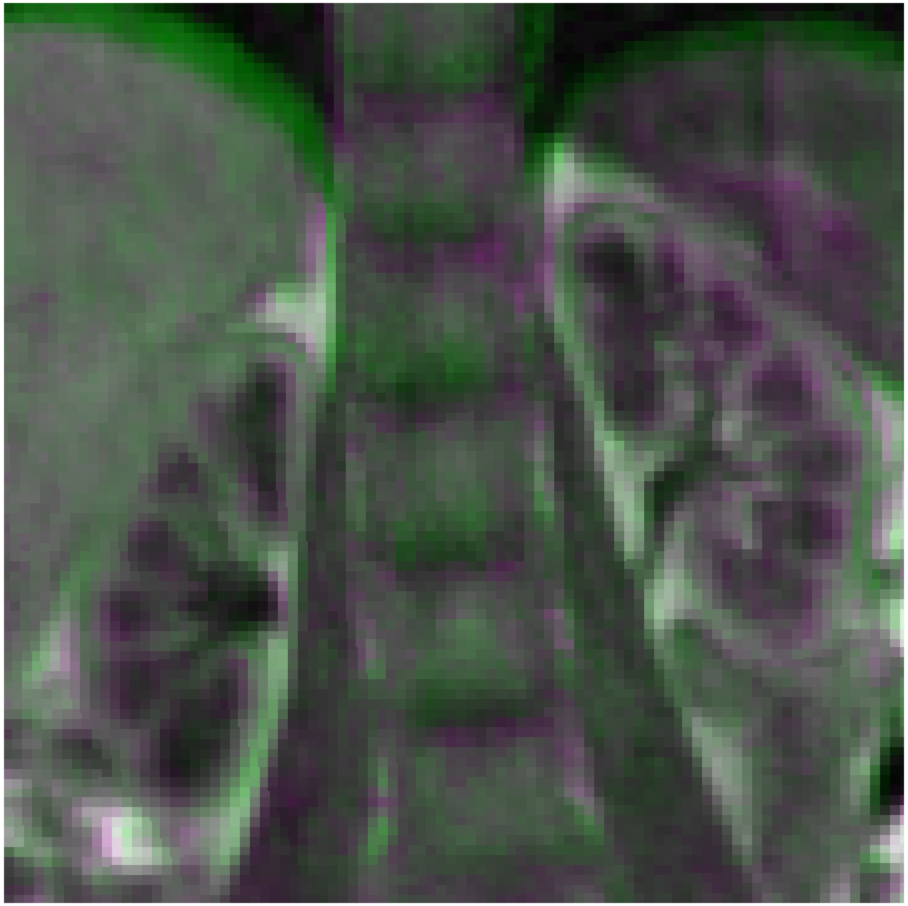


After

Applications



Before



After