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Excercise 3
Reflections in Long Lines

Aim of the exercise

The aim of the exercise is to get acquainted with the wave phenomena in the transmission lines with the properties of the long line, and the methods of measuring the long line parameters and its load.

Introduction

If we have the circuit with extremely low frequency electromagnetic field, the delay effects resulting from the finite velocity of EM wave propagation can be omitted. Such circuit can be presented as a concentrated parameters circuit. The current and voltage characteristics in such circuit are the time functions. If the phenomena resulting from limited velocity of the EM wave are equivalent in the time domain to the forcing factor in the circuit, then the electric and magnetic quantities should be considered as the two variables function – time and place.
Long lines are defined as the electric lines, where there exists the influence of the line parameters distribution on the current and voltage characteristics. If the parameters are uniformly distributed along the line, then such line is called uniform long line. The length of the long line is approximately equal to the wavelength of the transmitted signal.

Wave phenomena in the long line

The infinitely short segment of the long line can be modeled using the equivalent circuit presented in fig. 1.

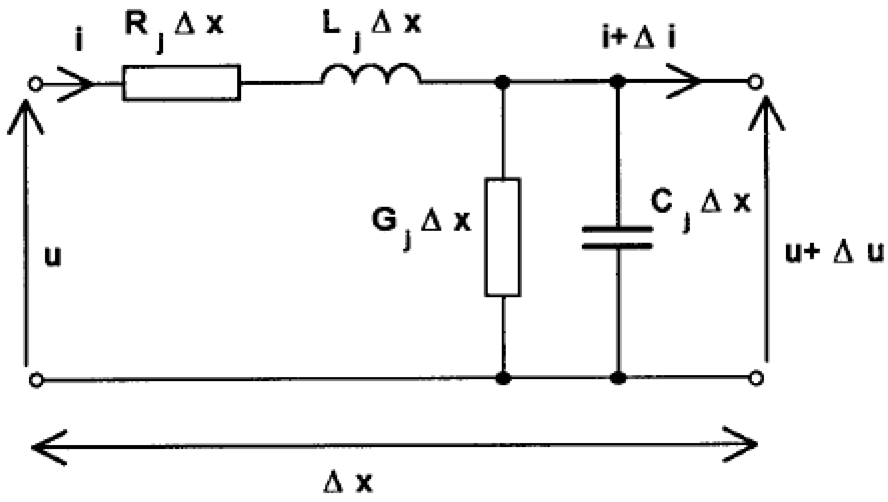


Fig. 1 The schematic of the long line segment using concentrated parameters.

where:
R_j – unit resistance of the line;
G_j – unit conductance;
L_j – unit inductance;
C_j – unit capacitance;

For the line segment of length Δx the current and voltage characteristics are expressed by the following equations:

$$-\frac{\partial u}{\partial x} = R_j i + L_j \frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial x} = G_j u + C_j \frac{\partial u}{\partial t}$$

We limit our considerations to the lossless line. In this case R_j=0 and G_j=∞.

Consider the circuit presented in fig. 2, which will be examined in the practical part of this exercise. The long line of length l, unit inductance L_j and unit capacitance C_j, which has no initial energy, has the load Z_2 connected to its end and is supplied with the generator of internal impedance Z_1 and electromotive force e(t) of value 0 for t>0.

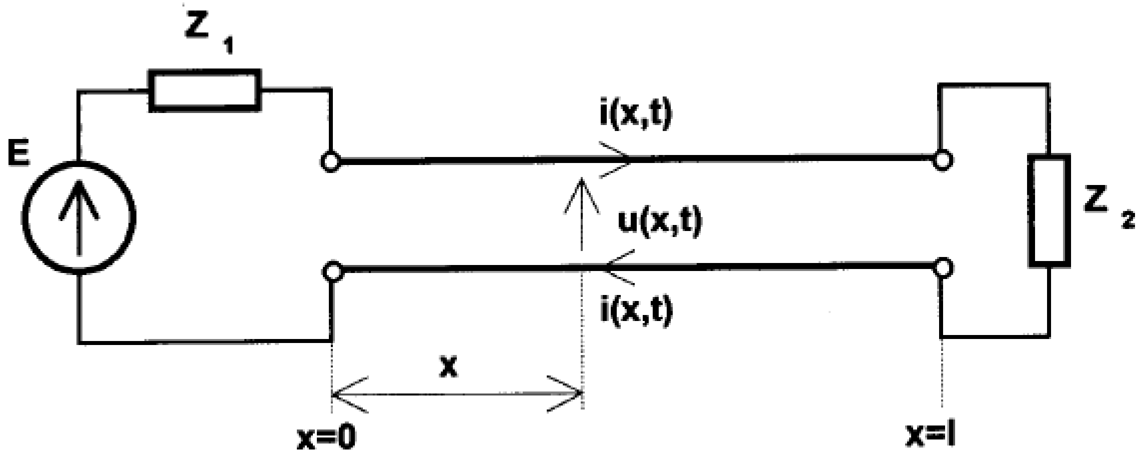


Fig. 2. Laboratory arrangement with the long line.

The laboratory arrangement presented in fig. 2 can be described by the following equations:

$$U(x,s) = \frac{E(s)R_0}{Z_1 + R_0} \cdot \frac{e^{-s\frac{x}{v}} + \Gamma_2 e^{-s\frac{21-x}{v}}}{1 - \Gamma_1 \Gamma_2 e^{-s\frac{1}{v}}} \qquad (3)$$

$$I(x,s) = \frac{E(s)}{Z_1 + R_0} \cdot \frac{e^{-s\frac{x}{v}} - \Gamma_2 e^{-s\frac{21-x}{v}}}{1 - \Gamma_1 \Gamma_2 e^{-s2\frac{1}{v}}} \qquad (4)$$

where:

s – complex variable;

U(x,s) and I(x,s) – Laplace transforms of voltage and current at the distance x from the beginning of the line;

E(s) – Laplace transform of the electromotive force of the generator;

$R_0 = \sqrt{\frac{L_0}{C_0}}$ – characteristic resistance of the line;

$v = \frac{1}{\sqrt{L_0 C_0}}$ – velocity of wave propagation along the line;

$\Gamma_1 = \frac{Z_1 - R_0}{Z_1 + R_0}$ – reflection coefficient at the beginning of the line;

$\Gamma_2 = \frac{Z_2 - R_0}{Z_2 + R_0}$ – reflection coefficient at the end of the line;

Equations (3) and (4) in the range 0<x<1 can be represented in the form of the following sequences:

$$U(x,s)=\frac{E(s)R_0}{Z_1+R_0}\left[e^{-s\frac{x}{v}}+\Gamma_2 e^{-s\frac{2l-x}{v}}+\Gamma_1\Gamma_2 e^{-s\frac{2l+x}{v}}+\Gamma_1\Gamma_2^2 e^{-s\frac{4l-x}{v}}+\ldots\right] \quad (5)$$

$$I(x,s)=\frac{E(s)}{Z_1+R_0}\left[s^{-s\frac{x}{v}}-\Gamma_2 e^{-s\frac{2l-x}{v}}+\Gamma_1\Gamma_2 e^{-s\frac{2l+x}{v}}-\Gamma_1\Gamma_2^2 e^{-s\frac{4l-x}{v}}+\ldots\right] \quad (6)$$

The elements of the sequence represent the voltage and current waves generated as a result of multiple reflections in the long line. The transforms of the voltage waves are described y the equations:

$U_0=\frac{E(s)R_0}{Z_1+R_0}$ – the transform of the primary voltage wave;
$U_1 = U_0\Gamma_2$ – transform of the 1st reflected voltage wave;
$U_2 = U_0\Gamma_1\Gamma_2$ – transform of the 2nd reflected voltage wave;
$U_3 = U_0\Gamma_1\Gamma_2^2$ – transform of the 3rd reflected voltage wave;

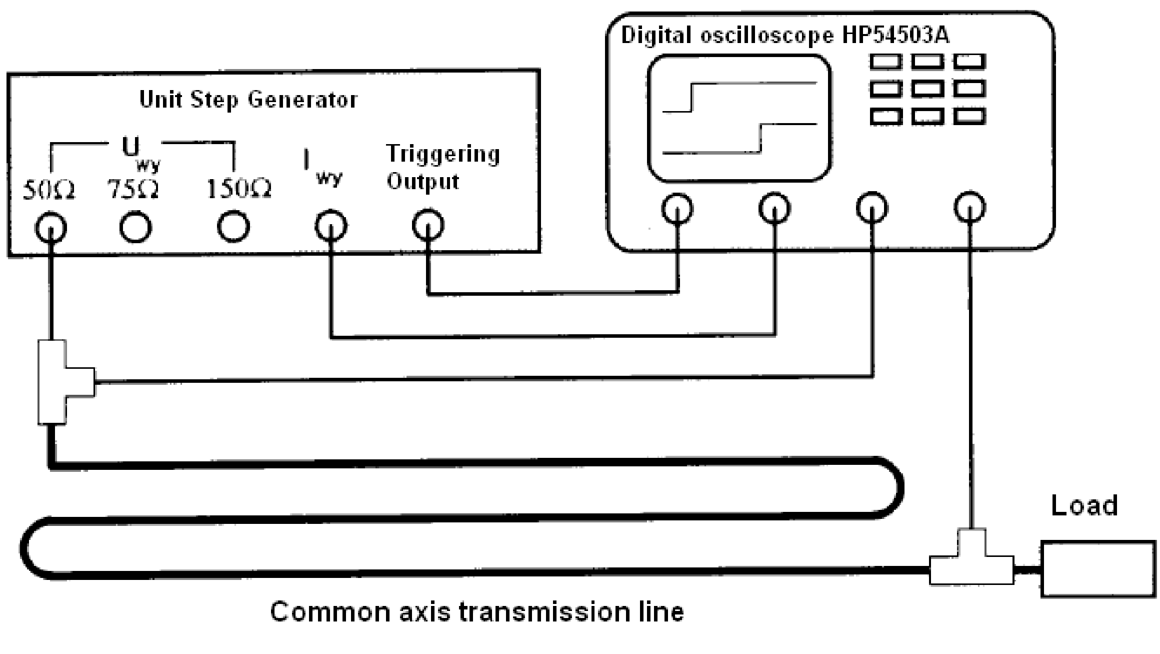
The voltage at the beginning and at the end of the transmission line can be found by performing the inverse Laplace transform for equations (5) and (6):

$$u(0,t)=u_0(t)+u_1\left(t-\frac{2l}{v}\right)+u_2\left(t-\frac{2l}{v}\right)+u_3\left(t-\frac{4l}{v}\right)+\ldots \quad (7)$$

$$u(1,t)=u_0\left(t-\frac{l}{v}\right)+u_1\left(t-\frac{l}{v}\right)+u_2\left(t-\frac{3l}{v}\right)+u_3\left(t-\frac{3l}{v}\right)+\ldots \quad (8)$$

where u_0, u_1, u_2… are the inverse transforms of U_0, U_1, U_2…
The same rules are for current waves.

Equipment



- unit step generator with output resistance $R_1 = 50\Omega$, 75Ω and 150Ω, triggering output and additional output for observing the output current from the generator (for 50Ω resistance);
- digital oscilloscope HP54503A;
- low-loss common-axis transmission line with characteristic resistance $R_0 = 50\Omega$ and length l=50m;
- low-loss common-axis transmission line with characteristic resistance 50Ω and unknown length;
- set of loads Z_2: matching resistance, short-circuit, open-circuit, capacitive load (parallel connection of capacitor and resistor) and inductive load (serial connection of inductor and resistor).

Measurement procedure

1) Build the measurement set as in fig. 3. Set the following generator and oscilloscope parameters: square wave of frequency equal to 100kHz. Minimum voltage value is set to 0V. Maximum voltage value is 1V. Set the sensitivity of the oscilloscope inputs to 0.2V/div and time scale to 50ns/div. Triggering in the oscilloscope should be completed by the channel not used for observations. Set the rising edge of the impulse so that you can observe the characteristics on the generator output as the unit impulse with amplitude 1V.
2) Measure the propagation time along the line for the short-circuited transmission line. Calculate the velocity v of the wave propagation assuming that l=50m. Connect the line segment of the unknown length. Calculate the length of this line based on the observed the propagation time and assuming that all the remaining parameters of both lines are the same.
3) Observe the voltage characteristics at the beginning and at the end of the transmission line and the current characteristics at the beginning of the transmission line supplied from the generator with impedance equal to the wave impedance of the transmission line. The line is:
   a) short-circuited at the end;
   b) open-circuited at the end;
   c) ended with matching resistance;
   d) with conductive load;
   e) with inductive load.

   Include the characteristics in the report. Observe the influence of the frequency on the shape of the characteristics. Increase the frequency to 1MHz.
4) Examine the voltage characteristics at the end of the transmission line and estimate the values of resistance, capacitance and inductance of the loads of capacitive and inductive type respectively. The matching of the impedance of the transmission line from the transmitter side will eliminate the multiple reflections of the wave. It will facilitate the analysis of the characteristics. In this case we have:

   $$e(t) = 1(t) \Rightarrow E(s) = \frac{1}{s}, \Gamma_1 = 0, \Gamma_2 = \frac{Z_2 - R_0}{Z_2 + R_0}, U_0 = \frac{1}{2s}, U_1 = U_0\Gamma_2, U_2 = 0, U_3 = 0...$$

   For the long line loaded by the parallel connection of resistor R and capacitor C:

$$Z_2(s)=\frac{R}{1+sRC},\Gamma_2=1-2\frac{s+\frac{1}{RC}}{s+\frac{1}{R_0C}+\frac{1}{RC}},U_1=\frac{1}{2}E(s)-\frac{s+\frac{1}{RC}}{s+\frac{1}{R_0C}+\frac{1}{RC}}E(s)$$

$$u(l,t)=1\left(t-\frac{l}{v}\right)\frac{R}{R+R_0}\left[l-e^{-\frac{R+R_0}{RR_0C}\left(t-\frac{l}{v}\right)}\right]$$

For the long line loaded by the series connection of resistor R and inductor L:

$$Z_2(s)=R+sL,\Gamma_2=\frac{R-R_0+sL}{R+R_0+sL},U_1=\frac{1}{2}\left[\frac{R-R_0}{s\left(s+\frac{R+R_0}{L}\right)}+\frac{1}{s+\frac{R+R_0}{L}}\right]$$

$$u(l,t)=1\left(t-\frac{l}{v}\right)\frac{1}{R+R_0}\left[R+R_0e^{-\frac{R+R_0}{L}\left(t-\frac{l}{v}\right)}\right]$$

5) Observe the voltage and current characteristics in not matched line at the input and at the output.
   a) $R_1 = 150\Omega$, $Z_2 = \infty$,
   b) $R_2 = 150\Omega$, $Z_2 = 0\ \Omega$.

   Compare obtained plots with theoretical ones. Repeat the observations for the frequency of 1MHz when we have multiple reflections of square wave.
6) Measure the impedance of the loads using the vector analyzer and compare obtained results with those estimated in point 3.

Bibliography

[1] "Technika impulsowa", S. Sławiński, WNT 1973
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